

Analysis of physical processes via imaging vectors

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Abstract Practically, all modeling processes in one way or another are random. The foremost formulated theoretical foundation embraces Markov processes, being represented in different forms. Markov processes are characterized as a random process that undergoes transitions from one state to another on a state space, whereas the probability distribution of the next state depends only on the current state and not on the sequence of events that preceded it. In the Markov processes the proposition (model) of the future by no means changes in the event of the expansion and/or strong information progression relative to preceding time. Basically, modeling physical fields involves process changing in time, i.e. non-stationary processes. In this case, the application of Laplace transformation provides unjustified description complications. Transition to other possibilities results in explicit simplification. The method of imaging vectors renders constructive mathematical models and necessary transition in the modeling process and analysis itself. The flexibility of the model itself using polynomial basis leads to the possible rapid transition of the mathematical model and further analysis acceleration. It should be noted that the mathematical description permits operator representation. Conversely, operator representation of the structures, algorithms and data processing procedures significantly improve the flexibility of the modeling process.

1. Introduction

The following research is devoted to the modeling of physical processes. Up-dated mathematical modeling of physical processes implies solving the following tasks: 1) mathematical model description of a specific physical process (or set of processes); 2) algorithm formulation in solving this or that problem; 3) numerical algorithmic mapping on computing system architecture. All these tasks are related. Before proceeding in examining closely any physical natural process via mathematical methods it is necessary to identify the basic principles and dynamics enabling to describe its



quantitative and qualitative behavior adequately and simply i.e. produce a model. Real behavior of natural physical processes is actually much more complex comparable to primitive objects which are explorable by methods of modern theory. At the present development stage of Geoscience there are existing sets embracing numerous disordered elements. It is practically impossible to describe all these elements related to different Geoscience domains. To solve the tasks associated with ultra-long period processes, newly-developed methods in modeling and analysis of experimental data are required. Practically, all modeling processes in one way or another are random. Problem-solving of any random process is rather difficult. However, comparatively simple calculation methods are accessible only if general random processes are not considered, and, accordingly, only those processes having certain specific properties and being of some practical importance are considered. Such processes are termed as Markov processes [1]. Markov processes are one of the most important models for real existing natural processes, the apparatus of which is well-developed. The characteristic feature of Markov processes is the fact that the probability distribution of the next state depends only on the current state and not on the sequence of events that preceded it. Markov process of proposition (model) by no means changes in the event of the expansion and/or strong information progression relative to preceding time.

2. Theoretical background

The analysis of dynamic processes occurring under the influence of different fields could be based on various mathematical models. When modeling physical structures with time-evolving processes, the description of such processes should be correlated to an appropriate mathematical apparatus. Numerous characteristic features of the processes in a physical environment (system) could be attributed to the peculiarities of the stationary system itself. The parameters and structures of this system are all assumed to be constant in time. In this case, the direct model is applied based on the following statistic methods: regression, factorial analysis, argument-dependent method and etc. In the event of dynamic processes differential equations with constant coefficients are included. Nevertheless, there is on problem class which excludes these methods as inappropriate. These problems are attributed to the manifestation of explicit temporal and structured non-stationarity. In particular, the changing parameters of a physical environment (system) in time could result in the change of the structure itself furnishing the inapplicability of a mathematical apparatus, which, in its turn, is intended to describe a system with constant parameters and structures.

On the background of steady stationary state, such systems and phenomena develop slowly, and under insignificant exposure could sharply change. In this case, this development process evidently transforms to another stage, and, as a result, its parameters also change. As it has been mentioned all initially employed description methods, oriented on the stationary state, decrease the process adequacy and become inappropriate in describing any phenomena. To preserve the model adequacy of the process being described it is necessary to change the mathematical framework of basic unit model representation. The mathematical framework change poses a series of questions which could exclude further process development modeling.

One question is - the mathematical, algorithmic and software consistency level of modeling and data representation. It seems the modeling of any physical process, all data, including even initial data in modeling and/or design, is various, and, in this case, requires identically diversified structures for its representation. Such a diversity of structure data is generated not only by its source, but also by its possible integrated storage, processing and representation. To solve the whole complex of above-mentioned problems a properly selected mathematical apparatus could serve as a base which is

completely adequate for the described physical processes, on the one hand, and could provide uniform procedure of process modeling and data representation, on the other hand.

In our opinion, applied functional analysis could be a mathematical apparatus. Based on this fact, the designed model system architecture possesses sufficient general and/or universal properties to establish a theoretical model, their algorithms and accompanying software. The rather extensive universality of this proposed method is based on the fact that the methods being generated within the functional analysis could be used to describe not only the modeling process itself, but also data processing in general. Conversely, all descriptions presume operator representation. Further, operator representation of structures, algorithms and data processing significantly improve the modeling flexibility. Moreover, due to the uniform representation of model components as array structures and signals as vectors, the modeling system becomes more reliable and universally accepting block-oriented description in the form of series-parallel block diagrams.

The most attractive examples of the above-described approach are such methods as spectrum estimation of non-stationary systems [2], interpolation method [3], and method of imaging vectors [4]. Data processing of both stationary and non-stationary cases are easily interpreted by the above-mentioned methods. Moreover, these methods can be applied in describing operations on discrete variables. It is possible to determinate precisely the problem formulation which is acceptable for using random variables within the spectrum method itself and its supporting methods. This aspect considers practical requirements when referring to random factors and processes which develop on the background of strictly specified regularities.

When considering physical fields modeling is of great importance due to the fact that processes changing in time, i.e. nonstationary processes are mainly involved. In this case, the application of Laplace transformation provides unjustified description complications. Transition to other possibilities results in explicit simplification. One possibility is the application of the functional analysis. The method of imaging vectors provides constructive mathematical models with subsequent changes in modeling and analysis. The flexibility of the model itself using polynomial basis leads to the possible rapid transition of the mathematical model and further analysis acceleration.

The method is based on the principle of the transition of mathematical operations to operation coefficients representing functions and their derivatives and integrals. This is particularly effective in linear cases where existing mathematical support and software could be applied. The operation is based on the representation of the function involving generalized Fourier constants in polynomial basis:

$$f(t) = \sum f_i P_i(t), \quad (1)$$

where, $(P_i(t))$ is function representation basis

Representing polynomial $(P_i(t))$ is included in the polynomial basis and is enormously flexible, independently and selectively, i.e. they are easily normalized and adapted to an orthogonality interval. Due to this property the analysis could be repeated, which, in its turn, is convenient in modeling.

Vector coefficients

$$F = \{f_0, f_1, \dots, f_{n-1}\}, \quad (2)$$

are termed as imaging vector of the function in basis $(P_i(t))$.

In the transition to different operations involving differentiation and spatial integration of their equivalents, the coefficients are vector-matrix expressions [4]. Particularly, such expressions are various differential-integral equations. These expressions could be the problem basis in considering

the modeling of physical processes, including initial and boundary non-stationary conditions. The data representation is exceptionally similar to computerization and programming support (software support).

Practically, all modeling processes in one way or another are random. The foremost formulated theoretical foundation embraces Markov processes, being represented in different forms. It is critically important that this property is easily extended to software-based and mathematical model properties, which are considered with imagining vector method application.

Assuming that the operation is described by linear (the simplest examples) equation systems [2], including initial conditions:

$$\frac{dP}{dt} = \begin{bmatrix} P_{11}(t) & \dots & P_{1n}(t) \\ \dots & \dots & \dots \\ P_{N1}(t) & \dots & P_{NN}(t) \end{bmatrix} \begin{bmatrix} P_1(t) \\ P_2(t) \\ \cdot \\ \cdot \\ P_N(t) \end{bmatrix} \quad P = \begin{bmatrix} P_1(t) \\ P_2(t) \\ \cdot \\ \cdot \\ P_N(t) \end{bmatrix} \quad (3)$$

$P_{li}(t)$ – probability state, $P_{ij}(t)$ – transition probability from the state S_i to the state S_j .

This system of linear differential equations is consistent to the algebraic equation system, where the argument t is the matrix J . Therefore, we have a block matrix of imagining matrix of coefficients:

$$\{P_{ij}(t)\} \rightarrow \{P_{ij}(J)\}, \quad (4)$$

while, vector function image is a block vector:

$$\left\{ \begin{array}{l} P_1(t) = \pi_{10} \overrightarrow{P_0}(t) + \pi_{11} \overrightarrow{P_1}(t) + \pi_{12} \overrightarrow{P_2}(t) + \dots + \pi_{1N-1} \overrightarrow{P_{N-1}}(t) \\ P_2(t) = \pi_{20} \overrightarrow{P_0}(t) + \pi_{21} \overrightarrow{P_1}(t) + \pi_{22} \overrightarrow{P_2}(t) + \dots + \pi_{2N-1} \overrightarrow{P_{N-1}}(t) \\ \cdot \\ \cdot \\ P_N(t) = \pi_{N0} \overrightarrow{P_0}(t) + \pi_{N1} \overrightarrow{P_1}(t) + \pi_{N2} \overrightarrow{P_2}(t) + \dots + \pi_{NN-1} \overrightarrow{P_{N-1}}(t) \end{array} \right\}, \quad (5)$$

$$P_1(t) = \begin{pmatrix} \pi_{10} \\ \pi_{11} \\ \pi_{12} \\ \cdot \\ \cdot \\ \pi_{1N-1} \end{pmatrix} = \pi_1, \quad P_2(t) = \begin{pmatrix} \pi_{20} \\ \pi_{21} \\ \pi_{22} \\ \cdot \\ \cdot \\ \pi_{2N-1} \end{pmatrix} = \pi_2 \dots P_N(t) = \begin{pmatrix} \pi_{N0} \\ \pi_{N1} \\ \pi_{N2} \\ \cdot \\ \cdot \\ \pi_{NN-1} \end{pmatrix} = \pi_N. \quad (6)$$

This algebraic system of equations could be solved.

Noting the linear equations, the spectrum method enables to write the expression of closed-form equation solutions using matrix symbolism. The application of the spectrum method results in the algebraic connection between the system attributes. This connection is expressed as an operator matrix form. In this case, it is possible to obtain an expression of transfer functions, system output signals and their explicit characteristics; perform conversion symbolically describing the investigated system to simplify the expressions; and in the final investigation stage of finding the variable characteristics to perform computations according to the matrix-operator formula by general-purpose subroutines and reference sources.

Spectrum method used for system solution of differential equations has the following advantages: 1. homogeneous representation for operations and procedures; 2. homogeneous representation of one-dimensional signals; and 3. possibility of structure determination.

The following fact should be highlighted- a set of conditions associated with the loosening of time-dependency models are reduced to simple parametric associations within the operator representation domain. As a result, there is the possibility of not only solving broad-class problems, but also accumulating information.

It is possible to determinate precisely the problem formulation which is acceptable for using random variables within the spectrum method itself and its supporting methods. This aspect considers practical requirements when referring to random factors and processes which develop on the background of strictly specified regularities.

Block diagrams are introduced to solve complicated non-stationary linear systems, as in the case of determination analysis. The first stage involves the differential equations connecting input and output systems [4]. In this case, the apparatus of linear differential operators is used. The second stage involves the transition to spectrum characteristics. In this case, the transition is based on spectrum (matrix) description of the systems and adapted to computer software.

3. Conclusion

The problem-solving of any random process is rather difficult. However, comparatively simple calculation methods are accessible only if general random processes are not considered, and, accordingly, only those processes having certain specific properties and being of some practical importance are considered. Such processes are as Markov processes [1]. The characteristic feature of Markov processes is the fact that the probability distribution of the next state depends only on the current state and not on the sequence of events that preceded it. Markov processes involve a well-developed theoretical foundation.

Considering Markov processes changing in time the possibility of state transition could indicate specific investigation methods: method of imaging vectors and spectrum method. Both methods are oriented in investigating orthogonal functional basis within an energy-limited function which

corresponds to the physics of obtained results and introduces a special operator expression describing the physical phenomenon in finite time interval. The behavior of resulting relations is that matrix representation of linear operators is used as an information carrier of the processes. In these cases the introduction of numerical analysis modeling into up-dated computer programs is possible.

References

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